Appendix B: Alternative Utility Functions and Bid-Ask spreads

In the main body of the paper, I derive expressions for bid-ask spread using a Constant Relative Risk Aversion (CRRA) utility function for all market participants. In this appendix, I derive expressions for the bid-ask spread using Constant Relative Risk Aversion (CRRA) utility and show the results are qualitatively similar.

Consider the same game-theoretic setup as described in the main body of the paper. However, now players have utility function:

The final wealth of the liquidity trader is given by:

Let be the expected wealth. A second-order Taylor series approximation of the utility function around is given by:

Then

Where I have used

Since . We have:

where I write to emphasize that the liquidity trader’s expected wealth is a function of the liquidity trader’s order quantity. To find the optimal order quantity, we can set the derivative of the expected utility to zero:

We can solve this to obtain:

This is like the optimal order quantity we derived under CARA

With an additional term in the numerator, showing the order quantity is sensitive to the expected level of wealth of the trader.

We can similarly derive the reservation prices for the dealers. For simplicity, I work through the reservation bid price for dealer 1.

If dealer 1 receives a random inventory position , . If s/he posts the best bid price, his/her wealth at the end of the period is:

If s/he does not trade, his/her wealth will be:

At some price the dealer will be indifferent between trading and not trading. This will occur when .

Let be the expected wealth

The second order Taylor-series expansion about is:

Taking expectations:

Using the fact that we obtain:

We can similarly take the Taylor Series approximation of the utility function at the no-trade level of wealth:

We can substitute and into the expressions above and set . We obtain the reservation price:

We cannot solve for explicitly, but note it has a similar form to the CARA reservation price.

In competitive markets, the dealer’s markup will tend to zero and prices will tend to the reservation prices.

Thus we find that using CRRA utility function (or with an utility function which is at least as concave as a CRRA utility function), the optimal order quantity and bid-ask spreads are qualitatively similar to the CARA utility case, except quantities and spreads are sensitive to the level of expected wealth of the players.